Auctions and Optimal Bidding

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Agenda

- Examples of auctions
- Bidding in private value auctions
- Bidding with termination fees and toeholds
- Bidding in common value auctions
- Implications for takeovers

Auction types

- Ascending (English) bids
- Descending (Dutch) bids
- First-price
  - Winner pays own bid
- Second-price
  - Winner pays price equal to the second highest bid
- Open – all bids are observed by everyone
  - Typically first price
- Sealed bid – you observe only your own bid
  - Typically second price
- Seller’s reserve price – seller may refuse to sell at a prespecified minimum bid

Six auction games

I PRIVATE VALUE AUCTIONS
- 1. Private (uncorrelated) values
- 2. Private values with positive outside optio
  - termination fee
- 3a. Private values with negative outside option
  - industry fall-out
- 3b. Private values with strategic advantage
  - Toehold

II COMMON VALUE AUCTIONS
- 4. Common (correlated) values
  - the jar
- 5. Common values
  - wallet game
- 6. Common values with strategic advantage
  - wallet game

Game 1:
Private (uncorrelated) value auction

- Bidders attach a personal or private value v to the object being sold that is independent of everyone else’s valuation
- Examples:
  - A bottle of vintage wine to be consumed
  - A house that you are going to live in
  - A painting or piecen of art
  - A target where synergioes atre unique to each (strategic) bidder
- Second-price sealed bid auction

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Game 1:
Optimal bid: p=v

- Suppose p=v. Quit?
  - Yes. Bidding p>v implies a loss if you win and zero profit if you lose. So the expected value of continuing to bid is negative
- Suppose p<v. Quit?
  - No. Quitting now implies zero profit while continuing gives you the chance of winning and making a profit (as long as p<v). So the expected value of continuing to bid is positive
- ”Ratchet” solution. Highest (most efficient) bidder wins

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Revenue equivalence

- With private (unaffiliated) values, a first-price open ascending auction produces the same auction revenue for the seller as a second-price sealed bid auction
  - Assuming that bidders are risk-neutral, bidding costs are zero, etc.
- Intuition: lower valuation bidder will drop out at his or her own valuation

Game 2:
Private value w/positive outside option

- One bidder gets a compensation if losing (or quitting) the auction
  - If you win, you get the item
  - If you lose, you get a $ compensation (from auctioneer)
  - Example: a termination fee
- Second-price sealed bid auction

Game 2:
Optimal bid: p=v-(value of outside option)

- So the outside option leads to less aggressive bidding. Why?
- If you win the auction, you get v, but you also forego the break-up fee (f)
  - Quitting after a bid of v-f means you take the outside payment f
  - So, if you bid more than v-f, you will end up with a gain that is less than f
- Auction results in inefficient allocation of asset whenever $v_A - f < v_B < v_A$

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Game 3a:
Private value w/negative outside option

- One of you will have a negative payoff if you lose
  - If you win, you get the item
  - If you lose, you must pay (the auctioneer) a $ compensation
- Example: industry competition
  - If there is a shake-out in the industry and only the merged company will survive, loosing a competition for a target may imply a negative payoff
- Second-price sealed bid auction

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Game 3a: Optimal p=v+(value of outside option)

- So, a negative outside option (-n) leads to more aggressive bidding:
  - If you win the auction, you get v
  - If you loose you get –n
  - You are willing to bid until your gain from winning equals your gain from loosing, i.e. until b-v=n, or the optimal bid b=v+n
- Auction results in inefficient allocation of asset whenever $v_A<v_B<v_A+n$

Game 3b: Private value with advantage

- Variation on Game 3a
- Optimal bid: p=v+advantage
  - If you win the auction, you get v plus an advantage (a). If you bid more than v+a, you lose money
  - If you lose the auction, you get zero. So quitting at a bid less than v+a means leaving a possible gain on the table
- An advantage conditional on winning makes the bidder more aggressive, similar to a negative outside option

Toehold bidding

- Notation:
  - p denotes price paid if you win
  - t_W and t_L denote payoff on toehold if win or lose
  - Π_W and Π_L denote total payoff if win or lose
- $\Pi_W = v_1+t_W-p$ and $\Pi_L= t_L$ so $\Pi_W \geq \Pi_L$
- <u>Note</u>: $p \leq v_1+t_W- t_L$
- So, if $t_W=t_L$ then $p=v_1$ (no overbidding)
- If $t_W> t_L$ then $p>v_1$ (overbidding)
- $E(\Pi)$ = Prob(win)x Π_W + Prob(lose)x Π_L

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Bidding with toehold α

- If B1 wins, payoff is $v_1-(1-\alpha)p_2$ with prob. $G(p_1)$
- If B1 loses, payoff is αp_1 with prob. $1-G(p_1)$

$$E(\Pi_1) = v_1 G(p_1) - (1-\alpha)\int_0^{p_1} p_2 g(p_2)dp_2 + \alpha p_1[1-G(p_1)]$$

$$\boxed{p_1^* = v_1 + \alpha \frac{1-G(p_1^*)}{g(p_1^*)}}$$

For uniform distribution:

$$p_1^* = \frac{v_1+\alpha}{1+\alpha}$$

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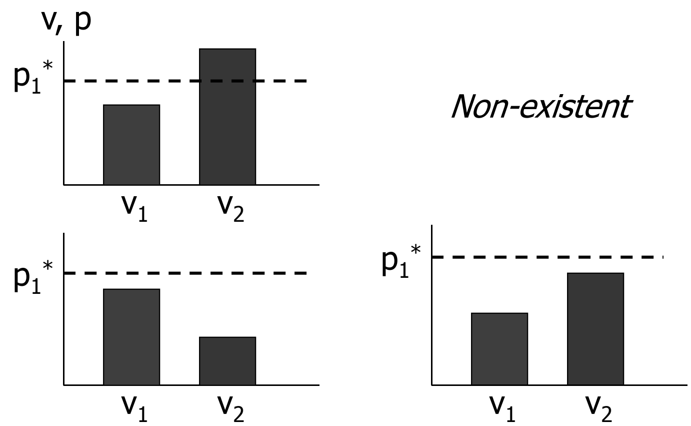
Bidder 1 overbids

Efficient outcomes | Inefficient outcomes

Bidder 1 loses auction

Bidder 1 wins auction



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II. Common value auctions

- The value of the item being sold is the same for all bidders
- Bidders have uncertain estimates of the common value
- Example:
  - Bidding for a jar of dollar bills
  - Financial bidders for a target

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Game 4:
Common value

- You are going to bid for the jar of dollar bills
- Open first-price auction
- Start by estimating the value of jar
- Hold your hand in the air
- I am the auctioneer
- Lower your hand when you want to drop out of the bidding

”Winner’s curse”

- Suppose that some estimates are above and some estimates are below the true value of the money jar
- The estimation error is the difference between the estimate and the true value
- If everyone bid their own estimate, the one with the largest positive estimation error wins
- Thus, winning is a ”curse”

Winner's curse: Example

- 4 bidders bid for an object with unknown true value of 25
- Their value estimates are:
  - v_1=10
  - v_2=20
  - v_3=30
  - v_4=40
- Here, the average estimate is 25
- If all bidders bid up to their estimate, bidder 4 wins, paying $30 (when bidder 3 drops out) and losing $5 on average

Winner's curse: Optimal bid strategy

- Highest bid: The price where you expect to make zero profit if you win
  - Compute the value of the target conditional on winning
  - Ignore the value of the target conditional on losing (for you, this value is zero)
  - This strategy leads to "bid shaving" in response to the winner's curse
  - You observe the number of bidders in the auction and learn from their bids

Winner's curse:
Example w/4 bidders cont'd

- Assume symmetric bid strategies:
  - Bidders use the same optimal bidding strategy as a function of their valuation
  - Thus, two bidders with the same value will quit from the action at the same price
- Assume that all individual estimates of v are independent of each other and drawn from the same distribution with mean (10+20+30+40)/4 = $25
- When is it optimal to drop out?

Winner's curse example:
B1 should drop out at $10

- B1 knows her own valuation: v_1=10
- If B1 wins at p=11, everyone else must have valuations ≤11
  - The expected value conditional on winning with a bid of 11 is E(V) ≤ (10+11+11+11)/4 = $10.75
  - So B1 expects to makes a loss with a bid of 11
- So, B1 should not bid above $10
  - It is suboptimal for B1 to quit at prices below $10
  - With a bid of 9, E(V) ≤ (10+9+9+9)/4 = $9.25

Winner's curse example: B2 should drop out at $17.50

- Suppose B2 bids 20. What is the expected value if B2 wins?
- E(V) ≤ (10+20+20+20)/4 = $17.50
  - You include the 10 since you know first bidder dropped out at 10
  - You assign 20 to everyone else since you assume that you win at that price
- Thus, even with a valuation of 20, it is optimal for B2 to offer no more than $17.50

Winner's curse: General insights

- For all but the bidder with the lowest valuation estimate, the optimal bid depends on the behavior of the other bidders (the number of bidders remaining in the auction)
- You drop out of the auction although you have not reached your full valuation and although you know that others have valuations greater than yours
- You end up shaving your bid so as to resolve winner's curse. Not shaving is ”behavioral”

Game 5 - Common value: The wallet game (two bidders)

- You are B1 and you know you have v_1 dollars in your wallet
- You are bidding for the <u>sum</u> of what is in the yours and B2's wallets
- First-price open auction
- The winner receives the combined wallet value (v_1+v_2)
  - The loser has a zero payoff (you are compensated for the wallet loss)
- What is optimal bidding strategy, bid for bid?

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Wallet game: Highest bid for bidder i: p_i=$2V_i$

- Key: You learn something about the common value from the other's bid
- B1 and B2 have v_1=\$5 and v_2=\$7 in their respective wallets
  - Suppose B1 starts with a bid of 1
  - B2 just learned that $v_1 \geq 1$
  - Since $v_2 \geq 1$, B2 can safely bid 2
  - B1 can safely bid 3, since she now knows that $v_2 \geq$ \$1 and she has at least 2 herself
  - ….
  - Suppose B2 has bid 10 (=$2v_1$). B1 drops out
    - Why? If B1 bids 11 and wins, she loses \$1
  - B2 wins the auction and pays \$10

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Game 6:
Wallet game with strategic advantage

- The game is as before, but one bidder also gets $1 (from me) if he wins the auction
  - Two bidders B1 and B2 will bid for their combined "wallet-value"
  - The winning bidder pays his/her bid and receives the combined value
  - If the bidder with the advantage wins, he also gets $1
  - It is known to both bidders who has the advantage

Game 6:
Advantage makes a BIG difference

- Let v_1= $8 and v_2 = $6, sum $14
- Give B2 the advantage of $1
- B2 wins the auction, although $v_2+1<v_1$
- Why?

Game 6:
Bid sequence with advantage

- Key: B1 does NOT learn anything about the common value from the bid sequence
- Suppose B1 starts with a bid of 1
  - B2 learns that $v_1 \geq 1$
- B2 counters with a bid of 2
  - Safe bid since B2 has $1 advantage.
  - However, B1 learns nothing about v_2 from this opening bid
- B1 bids 3
  - B2 learns that $v_1 \geq 3$
- B2 now bids 4
  - Safe bid since $3 plus $1 advantage = 4
  - However, B1 still learns nothing about v_1

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Game 6:
Bid sequence, cont'd

- B1 bids 5
  - B2 learns $v_1 \geq \$5$
- B2 bids 6
  - Safe bid since $5 plus $1 advantage = 6
  - B1 still learns nothing about v_2
- B1 bids 7
  - B2 learns $v_1 \geq \$7$
- B2 bids 8
  - Safe bid since $7 plus $1 advantage = 8
  - B1 still learns nothing about v_2
- B1 drops out and B2 wins. B2 pays 8 and receives 14+1=15, for a gain of 7

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Basic Insight:
(common value setting w/strategic advantage)

- The bidder with the advantage will keep raising his bid and win the auction even if his value estimate is lower than that of the competing bidder
- The bidder with the advantage will win the auction with certainty regardless of the size of the strategic advantage
  - A small advantage for one bidder can have a huge impact on the outcome and price realized in the auction
  - This also holds for a toehold

How to sell a company

- Auction versus negotiation
- How to commit to accept the highest bid and reject subsequent offers?
  - Breakup fees, lockup options
- Leveling the playing field
  - Sell a small toehold to the weaker bidder
  - Compensate a second bidder for entering the auction, e.g. a white knight